

1. State the definition for $\ln(x)$:

$$\ln(x) \equiv \int_1^x \frac{1}{t} dt \quad \text{for } x > 0$$

2. State the definition for The exponential function, e^x :

$y = e^x$ is the inverse of $y = \ln x$
 i.e. $y = e^x \iff \ln y = x$

Compute the following derivatives

$$3. \frac{d}{dx}(e^x + e^{3x})^4 = 4(e^x + e^{3x})^3 (e^x + e^{3x})' \\ = 4(e^x + e^{3x})^3 (e^x + 3e^{3x})$$

$$4. \frac{d}{dx} 3^x x^3 = (3^x)'(x^3) + (3^x)(x^3)' \\ = (\ln 3) 3^x x^3 + 3^x (3x^2) \\ = 3^x x^2 [x \ln 3 + 3]$$

$$5. \frac{d}{dx} \ln(5-x)^6 = \frac{d}{dx} 6 \ln(5-x) = 6 \frac{d}{dx} \ln(5-x) \\ = \frac{6}{5-x} (5-x)' = \frac{6}{x-5}$$

$$6. \frac{d}{dx} \ln\left(\frac{x^2 \sqrt{4x+1}}{(x^3+5)^3}\right) = \frac{d}{dx} \left[\ln x^2 + \ln \sqrt{4x+1} - \ln (x^3+5)^3 \right] \\ = \frac{d}{dx} \left[2 \ln(x) + \frac{1}{2} \ln(4x+1) - 3 \ln(x^3+5) \right] \\ = \frac{2}{x} + \frac{1}{2} \cdot \frac{4}{4x+1} - \frac{3 \cdot 3x^2}{x^3+5} = \frac{\frac{2}{x}}{\frac{4}{4x+1}} + \frac{\frac{2}{2}}{\frac{3}{x^3+5}} - \frac{\frac{9x^2}{x^3+5}}{\frac{9}{x^3+5}}$$

Compute the following integrals

$$7. \int \frac{e^{\sqrt{t}}}{\sqrt{t}} dt$$

$$u = \sqrt{t}$$

$$du = \frac{dt}{2\sqrt{t}}$$

$$\int 2e^u du = 2e^u$$

$$= 2e^{\sqrt{t}} + C$$

$$8. \int \frac{e^x}{e^x - 2} dx$$

$$u = e^x - 2$$

$$du = e^x dx$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$= \ln|e^x - 2| + C$$

$$9. \int \frac{1}{x^2 e^{\frac{2}{x}}} dx = \frac{1}{2} \int e^u du$$

$u = -\frac{2}{x}$
 $du = \frac{2}{x^2} dx$
 $\frac{du}{2} = \frac{dx}{x^2}$

$$10. \int_{-1}^2 \frac{x}{x^2 + 3} dx$$

$u = x^2 + 3$
 $du = 2x dx$
 $\frac{du}{2} = x dx$

$$\frac{1}{2} \int_{x=-1}^{x=2} \frac{du}{u} = \frac{1}{2} \left(\ln u \right) \Big|_4^7$$

$$= \frac{1}{2} (\ln 7 - \ln 4) = \frac{1}{2} \ln \frac{7}{4}$$

$$11. \int_0^3 2xe^{x^2} dx$$

$u = x^2$
 $du = 2x dx$

$$\int_{x=0}^{x=3} e^u du = \int_0^9 e^u du = [e^u] \Big|_0^9 = e^9 - 1$$

12. In 1980 the population of a town was 21,000 and in 1990 was 20,000. Assuming the population decreases continuously at a constant rate proportional to the existing population, estimate the population in the year 2100.

$$t=0 \text{ is } 1980$$

$$P = P_0 e^{kt}$$

$$P_0 = 21000$$

$$\text{for } t=10 \text{ (1990)}$$

$$20000 = 21000 e^{10k}$$

$$\frac{20}{21} = e^{10k}$$

$$\ln\left(\frac{20}{21}\right) = 10k$$

$$\frac{1}{10} \ln\left(\frac{20}{21}\right) = k$$

$$\text{so } P = 21000 e^{\ln\left(\frac{20}{21}\right) \frac{t}{10}}$$

$$\text{for } t=120 \text{ (year 2100)}$$

$$P = 21000 e^{\ln\left(\frac{20}{21}\right) \frac{120}{10}}$$

$$= 21000 \left(\frac{20}{21}\right)^{12}$$

$$= \boxed{11694}$$

13. Write an equation for the amount Q of a radioactive substance with a half-life of 30 days, if 10 grams are present when $t=0$.

$$Q = Q_0 \left(\frac{1}{2}\right)^{\frac{t}{30}}$$

$$Q = 10 \left(\frac{1}{2}\right)^{\frac{t}{30}}$$

or

$$Q = 10 e^{(\ln \frac{1}{2}) \frac{t}{30}}$$

14. The number of fruit flies increases according to the law of exponential growth. If initially there are 10 fruit flies and after 6 hours there are 24, find the number of fruit flies after t hours.

$$y = y_0 e^{kt}$$

$$y = 10 e^{kt}$$

$$\text{at } t=6$$

$$24 = 10 e^{6k}$$

$$2.4 = e^{6k}$$

$$\ln(2.4) = 6k$$

$$\frac{1}{6} \ln(2.4) = k$$

$$y = 10 e^{\ln(2.4) \frac{t}{6}}$$

or

$$y = 10 (2.4)^{\frac{t}{6}}$$